Roll No. ....   Total Pages : 3

BT–1/D-24   **41073**

MATHEMATICS–I

Paper–B24–BSC–107

Time Allowed : 3 Hours]   [Maximum Marks : 70

Note : All questions are compulsory. The question carrying **ten** marks in each unit shall have a choice in attempting any of the **one** option.

UNIT–I

1. Define the Beta function and its relation to the Gamma function. 2½

2. Solve $\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{x}}-e}{x}$. 5

3. Determine the volume of the solid generated by revolving the ellipse :

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ about the major axis.

Or

Calculate : (a) $\Gamma(-7/2)$ 10

(b) $B(7/2, -1/2)$.

P. T. O.

UNIT–II

4. Explain Fourier series and Euler's Formulae. 2½

5. Examine the convergence of the series :

$$\sum n!\frac{2^n}{n^n}.$$

6. Determine the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

Or

Test the convergence of the series : 10

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \frac{5^5 x^5}{5!} + \text{---}\infty.$$

UNIT–III

7. Define the Homogeneous function and explain Euler's theorem on Homogeneous functions. 2½

8. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, then find :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} \text{ at } (1, -1, 0).$$ 5

9. Use Taylor's series of two variables to expand $e^x \log(1+y)$ up to six terms in the Neighborhood of $(0, 0)$.

Or

If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$. Show that : 10

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

UNIT–IV

10. Give the value of $\int_0^1 \int_{x^2}^{2-x} dx\, dy$. 2½

11. Determine the area between the parabolas : 5

$y^2 = 4ax$ and $x^2 = 4ay$.

12. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$.

Or

Change the order of Integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ and hence evaluate the same. 10